

Intro to Repeated Measures ANOVA

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Two-sample t-test

A method for comparing the means of two populations

- Example: A firm has a generous but rather complicated policy concerning end-of-year bonuses for its lower-level managerial personnel. The policy's key factor is a subjective judgment of "contribution to corporate goals." To investigate whether gender plays a role in end-of-year bonuses, a personnel officer took samples of 12 female and 18 male managers. Their bonus data is expressed as a percentage of yearly salary.
- $H_0: \mu_F = \mu_M$ vs. $H_a: \mu_F \neq \mu_M$

Female Bonuses	Male Bonuses
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4
x_5	y_5
x_6	y_6
x_7	y_7
x_8	y_8
x_9	y_9
x_{10}	y_{10}
x_{11}	y_{11}
x_{12}	y_{12}
	y_{13}
	y_{14}
	y_{15}
	y_{16}

ANOVA (Analysis of Variance)

A method for comparing the means of three or more populations

- Example: In a survey to determine child care costs for working parents, the local Chamber of Commerce randomly samples licensed child care centers in four regions of the metropolitan area and obtains weekly costs of child care for a two-year old. The purpose of the survey is to see whether and how average child care expense varies according to region.
- $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ vs. $H_a: \mu_i \neq \mu_j$ for some $i \neq j$

ANOVA Data Layout

(Child Care Costs Example: $\bar{Y}_{..} = \$88.99$)

EastSub	Downtown	WestSub	SouthSub
90.00	94.00	82.00	86.50
87.50	97.50	84.50	88.00
89.50	94.00	88.00	89.50
90.00	92.50	85.50	85.00
$\bar{Y}_{1.} = 89.25$	$\bar{Y}_{2.} = 94.50$	$\bar{Y}_{3.} = 85.00$	$\bar{Y}_{4.} = 87.25$

Why is a technique for comparing **means** called "Analysis of Variance?"

- Using the context of the study on child care costs, we partition the total variability in weekly child care costs into two quantities: (1) that portion of the total variability in child care costs that can be explained by regional differences and (2) that portion of the total variability that is left unexplained.
- If quantity (1) is large compared to quantity (2), then we have evidence that the average child care costs for the different regions are not all the same.
- $SSTO = SSTR + SSE$
(1) (2)

Formulas for Sums of Squares (Child Care Costs)

$$SSTO = SSTR + SSE$$

- $SSTO = \sum_{i=1}^4 \sum_{j=1}^4 (Y_{ij} - \bar{Y}_{..})^2 = 243.64$

- $SSTR = \sum_{i=1}^4 \sum_{j=1}^4 (\bar{Y}_{i.} - \bar{Y}_{..})^2 = 195.31$

- $SSE = \sum_{i=1}^4 \sum_{j=1}^4 (Y_{ij} - \bar{Y}_{i.})^2 = 48.33$

Data layout and sampling for two-sample t-test and ANOVA

- There is only one measurement on each subject.
- The samples are independent.

Repeated Measures ANOVA

- All members of a random sample are measured under a number of conditions, yielding more than one measurement on each subject.
- Data layout: The columns of data are not independent of each other, but are related. Standard analysis of variance should not be applied.
- Repeated measures ANOVA is used for several reasons.

#1 Some research hypotheses require repeated measures

- Example: Eight 10-year old children were shown a cartoon and asked to rate the humor in the cartoon. They were shown the same cartoon two days later and then again two more days later. They rated the humor of the cartoon each time. The experimental question is whether ratings of humor change over time.

Data Layout for Repeated Measures ANOVA (Humor Data)

First Rating	Second Rating	Third Rating	Average Rating
6	5	2	$\bar{Y}_{1\cdot} = 4.33$
5	5	4	$\bar{Y}_{2\cdot} = 4.67$
5	6	3	$\bar{Y}_{3\cdot} = 4.67$
6	5	4	$\bar{Y}_{4\cdot} = 5.00$
7	3	3	$\bar{Y}_{5\cdot} = 4.33$
4	2	1	$\bar{Y}_{6\cdot} = 2.33$
4	4	1	$\bar{Y}_{7\cdot} = 3.00$
5	7	2	$\bar{Y}_{8\cdot} = 4.67$
$\bar{Y}_{\cdot 1} = 5.25$	$\bar{Y}_{\cdot 2} = 4.63$	$\bar{Y}_{\cdot 3} = 2.50$	$\bar{Y}_{\cdot\cdot} = 4.13$

Notes on the humor data

- Some subjects clearly think the cartoon is funnier than others. You can see this by looking at the subject averages shown in the right hand column. This variability reflects individual differences in humor.
- In repeated measures ANOVA, this source of variability is filtered out, so the variability in ratings of humor that is due to change over time can be detected.
- $SSTO = SSS + SSTR + SSE$

#2 When subjects are difficult to recruit, repeated measures designs are economical.

- In conducting clinical research, it is often difficult to find patients who have the condition to be studied *and* who are willing to participate in a clinical trial.
- In these situations, it's important to obtain as much information as possible once a suitable number of patients have been located.

- Example: Suppose three different compounds of blood-pressure-lowering drugs are being studied. A compound is administered to a patient during a given treatment period. After a sufficiently long "washout" period, another compound is given to the same patient. This procedure is repeated until the patient has been treated with all three compounds.
- It is crucial that the washout period between treatments is sufficiently long that the results from one compound would not affect the results for another compound. Also, the order in which the compounds are administered is randomized.

Caution: This type of design is applied in only those situations where the disease is relatively stable, so that following treatment and washout, each patient is essentially the same as prior to receiving treatment.

For example, we should expect that hypertension is stable enough that the patients would return to their predrug level blood pressures after washout of the first assigned compound before receiving the second assigned compound, and so on.

#3 When there is a great deal of variation between sample members, any difference in treatments may be difficult to detect. The differences between the individuals may mask any differences due to treatments. Repeated measures designs provide a way of accounting for this variation, and absorbing it so differences between treatment groups are apparent.

Example: National Paper Company must purchase a new machine for producing cardboard boxes. The company must choose between three machines. Since the machines produce boxes of equal quality, the company will choose the machine that produces the most boxes in a one-hour period. The company selected eight assembly workers to test the three machines. The number of boxes produced in an hour on each type of machine for each of the eight workers were measured.

Data Layout for Repeated Measures ANOVA (National Paper Company Example)

Operator	Machine 1	Machine 2	Machine 3	Operator Average
1	53	50	51	51.33
2	60	55	54	56.33
3	58	56	57	57.00
4	48	44	43	45.00
5	46	45	46	45.67
6	54	50	49	51.00
7	62	57	58	59.00
8	49	47	48	48.00

Notes on the National Paper Company data

- Some operators are clearly more experienced and skilled than others. You can see this by looking at the operator averages shown in the right hand column. Regular ANOVA doesn't account for this variation.
- In repeated measures ANOVA, this source of variability is filtered out, so the variability in number of boxes produced due to the different machines can be detected.

Row	NumBoxes	Machine	Operator
1	53	1	1
2	60	1	2
3	58	1	3
4	48	1	4
5	46	1	5
6	54	1	6
7	62	1	7
8	49	1	8
9	50	2	1
10	55	2	2
11	56	2	3
12	44	2	4
13	45	2	5
14	50	2	6
15	57	2	7
16	47	2	8
17	51	3	1
18	54	3	2
19	57	3	3
20	43	3	4
21	46	3	5
22	49	3	6
23	58	3	7
24	48	3	8

Regular ANOVA applied to National Paper data

- SSTO = total variation in number of boxes produced, and is partitioned into two components. SSTO = 667.3
- SSTR = that portion of the total variation in the number of boxes produced that can be explained by the difference in machines. SSTR = 52.3
- SSE = that portion of the total variation in the number of boxes produced that is left unexplained. SSE = 615.0
- P-value = .424. The differences between the machines is not uncovered. It is masked by the variation in the operators, which is not accounted for in regular ANOVA.

Repeated Measures ANOVA on National Paper

- SSTO = total variation in number of boxes produced, and is partitioned into **three** components. SSTO = 667.3
- SSS = that portion of the total variation in the number of boxes produced that can be explained by the difference in operators. SSS = 595.33
- SSTR = that portion of the total variation in the number of boxes produced that can be explained by the difference in machines. SSTR = 52.33
- SSE = that portion of the total variation in the number of boxes left unexplained. SSE = 19.67

Repeated Measures ANOVA on National Paper

- P-value < .001. The differences in the machines is detected this time. SSTO and SSTR are the same for both analyses, but the SSE values are different.
- In regular ANOVA, the variability in the number of boxes produced that is due to the different operators isn't accounted for, so it is dumped in with SSE (unexplained variation).
- In repeated measures ANOVA, the variability due to the different operators is accounted for, allowing us to detect the differences between machines

Formulas for Sums of Squares (National Paper Company)

$$SSTO = SSS + SSTR + SSE$$

$$\blacksquare SSTO = \sum_{i=1}^8 \sum_{j=1}^3 (Y_{ij} - \bar{Y}_{..})^2 = 667.3$$

$$\blacksquare SSS = \sum_{i=1}^8 \sum_{j=1}^3 (\bar{Y}_{i.} - \bar{Y}_{..})^2 = 595.33$$

$$\blacksquare SSTR = \sum_{i=1}^8 \sum_{j=1}^3 (\bar{Y}_{.j} - \bar{Y}_{..})^2 = 52.33$$

$$\blacksquare SSE = \sum_{i=1}^8 \sum_{j=1}^3 (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2 = 19.67$$